# Volterra-Wiener Functionals for Tasks Process Recognition Tasks Solving

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***Abstract*—This paper presents the nonlinear procedure based on Volterra-Wiener functional series. It could be used to synthesize decision functions in the recognition of processes for a broad class of stochastic inputs. It is shown the usage this nonlinear decomposition in recognition systems constructing.**

***Index Terms*—Decision functions synthesing, nonlinear signal decomposition, Volterra-Wiener functional, Wiener kernels measuring.**

## I. Introduction

The task of the recognition of processes can be defined as the statistical task of decision theory about presence one among of other alternate conditions. Thus, processing results of measurements gives us the accidental vector, whose components are all inspected parameters $x_1, x_2, \dots, x_p$, choosing in the correspondence with a defined criterion to one from alternative classes. Mathematically it can be presented as a system $F$ an input vector $\boldsymbol{x}$ to an output $y$:

$$y = F[\boldsymbol{x}], \tag{1}$$

i.e. in generally, this task consists to construct a model of the classifier $F$. A method based on the Wiener's theory of nonlinear systems allows to identify and to model systems (1) with the help of Volterra-Wiener functionals without additional a priori information about their structure [1], [2].

## II. Orthogonal Representation of the Nonlinear Systems

According to the Wiener's method the transformation F can be represented as a sum of orthogonal functionals $G_k[h_k, \mathbf{x}]$ [3]:

$$y \approx F[\boldsymbol{x}] = \sum_{k=0}^{S} G_k[h_k, \boldsymbol{x}], \tag{2}$$

where $\boldsymbol{x}$ is the vector of measurement results for the diagnosed system, $h_k$ are kernels of the Volterra-Wiener functionals. The functionals $G_k[h_k, \boldsymbol{x}]$ have to satisfy conditions of orthogonality:

$$\mathrm{M}\{ G_k[h_k, \boldsymbol{x}]\, G_n[h_n, \boldsymbol{x}]\} = \mathrm{M}\{( G_k[h_k, \mathrm{x}]^2\}, \quad k=n, \tag{3a}$$

$$\mathrm{M}\{ G_k[h_k, \boldsymbol{x}]\, G_n[h_n, \boldsymbol{x}]\} = 0, \; k \neq n, \tag{3b}$$

where M{·} is the operation of mathematical expectation. Using the minimum mean squared error criterion:

$$\mathrm{M}\{y - F[\boldsymbol{x}]\}^2 = \min,$$

as well as the conditions of the orthogonality(3_, 3b) for terms of the series(2):

$$\mathrm{M}\{ y \cdot G_k[h_k, \boldsymbol{x}]\} = \mathrm{M}\{( G_k[h_k, \boldsymbol{x}])^2\}, \; k=0,1, \dots ,S. \tag{3c}$$

As a system of orthogonal functionals $G_k[h_k, \boldsymbol{x}]$we consider the system of the polynomial functionals:

$$G_k[h_k, \boldsymbol{x}] = \sum_{n=0}^{k} \sum_{i_1=1}^{p} \dots \sum_{i_n=1}^{p} h_{kn}(i_1, i_2, \dots, i_n) x(i_1) \dots x(i_n), \quad (k=1 \dots S), \tag{4}$$

where $h_{kn}(i_1, i_2, \dots, i_n)$ are the kernels of the $k$-order polynomial, $i$ are the coordinate indexes of the vector of measurements $\boldsymbol{x}=(x_1, x_2, \dots, x_p)$.

It is possible to show that the following system of equations for the determination of the unknown polynomial kernels (4) $h_{k0}$, $h_{k1}(i_1)$, ... , $h_{kk}(i_1, i_2, \dots, i_k)$ under conditions (3_)-(3c) for $G_k[h_k, \boldsymbol{x}]$( $k=0,1, \dots ,S$) has to be solved:

$$\sum_{n=0}^{k} \sum_{i_1=1}^{p} \dots \sum_{i_n=1}^{p} h_{kn}(i_1, \dots, i_n) \{ ( \sum_{i_{n+1}=1}^{p} \dots \sum_{i_{n+k}=1}^{p} h_{kk}(i_{n+1}, \dots, i_{n+k}) \times \times \Gamma_{(n+k)x}(i_1, \dots, i_{n+k}) ) - \Gamma_{ynx}(i_1, \dots, i_n) \} = 0,$$

$$\sum_{n=0}^{k} \sum_{i_1=1}^{p} \dots \sum_{i_n=1}^{p} h_{kn}(i_1, \dots, i_n) \Gamma_{(n+m)x}(i_1, \dots, i_{n+m}) = 0, \quad (m=0,1,2, \dots, k-1), \tag{5}$$

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where $\Gamma_{(n+m)x}(i_1,\ldots,i_{n+m})$=M{$x(i_1)$ ... $x(i_n)$ $\cdot$ $x(i_{n+1})$ ... $x(i_{n+m})$}, $\Gamma_{ynx}(i_1,\ldots,i_n)$= M{$y$ $\cdot$ $x(i_1)$ ... $x(i_n)$}. Deciding (5) for $k$=0,1,...,$S$, all members $G_k[h_k,\boldsymbol{x}]$of the orthogonal representation (2) can be constructed.

## III. Results of Computer Experiment

For illustration of the above mentioned approach to solving problem of the recognition the following task has been solved [4]. The learning sample (Table 1) consists of 31 vectors is given, besides each vector has 12 coordinates representing 1 or 0. Each vector corresponds to the number 1 (the diagnosed system is in the state 1) or 2 (the diagnosed system is in the state 2). It is necessary to determine what state the system occurs, if the vector (see Table 2) is observed on its control output (since the sample is control, a priori it is known, the such output vector corresponds of the system in the state 2).

Let us assume that the series (2) consists of two functionals $G_0[h_0,\boldsymbol{x}]$and $G_1[h_1,\boldsymbol{x}]$, i.e.:

$$y \approx F_1[\boldsymbol{x}] = G_0[h_0,\boldsymbol{x}] + G_1[h_1,\boldsymbol{x}]. \quad (6)$$

TABLE I
THE LEARNING SAMPLE

| – | Coordinates of the learning vector $\boldsymbol{x}$ | | | | | | | | | | | | $y$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| 2 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 3 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| 4 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| 5 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 6 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 7 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 8 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 |
| 9 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 10 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 |
| 11 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 12 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 13 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 14 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 |
| 15 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 16 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 17 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 18 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 2 |
| 19 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 2 |
| 20 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 2 |
| 21 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 2 |
| 22 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 2 |
| 23 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 2 |
| 24 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 2 |
| 25 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 2 |
| 26 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 2 |
| 27 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 2 |
| 28 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 2 |
| 29 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 2 |
| 30 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 2 |
| 31 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 2 |

TABLE II
THE CONTROL VECTOR

| – | Coordinates of the control vector $\boldsymbol{x}$ | | | | | | | | | | | | $y$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
| 32 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 2 |

As it follows from (4) the system of the equations (5) for the functional $G_0[h_0,\boldsymbol{x}]$ can be described in the form:

$$h_{00}=\mathrm{M}\{y\}, \quad (7a)$$

and for the functional $G_1[h_1,\boldsymbol{x}]$:

$$\Gamma^0_{y1n}(i_1) = \sum_{i_2=1}^{p} h_{11}\Gamma^0_{2x}(i_1,i_2), \quad (7b)$$

$$h_{10} = -\sum_{i_1=1}^{p} h_{11}(i_1)\mathrm{M}\{\mathrm{x}(i_1)\}, \quad (7c)$$

where $\Gamma^0_{y1x}(i_1)$= M{$y$ $\cdot$ [$x(i_1)$ - M{x($i_1$)}]}, $\Gamma^0_{2x}(i_1,i_2)$ = M{[x($i_1$)- M{x($i_1$)}] [x($i_2$)- M{x($i_2$)}]}. Solving the equations (7_)-(7c), the unknown kernels of polynomials $h_{00}$, $h_{10}$, $h_{11}(i_1)$ can be found.

Thus, it has been obtained $h_{00}$=1,4194 , $h_{10}$= -0,4504 . The evaluation results for the first-order kernel $h_{11}(i_1)$ are represented in Table 3.

Then the control vector (Table 2) has been sent on an input of the designing system $F_1[\boldsymbol{x}]$, and in result, $F_1[\boldsymbol{x}_{32}]$= 2,2453 has been obtained from its output. Thus, if the vector of control parameters $\boldsymbol{x}_{32}$ is observed on the output of diagnosed system , then given system is in the condition 2 that corresponds to the condition of the task for the control vector.

TABLE III
THE EVALUATION RESULTS FOR THE FIRST-ORDER KERNEL

| $i_1$ | $h_{11}(i_1)$ |
|---|---|
| 1 | 0,1219 |
| 2 | 0,0606 |
| 3 | 0,1900 |
| 4 | 0,5957 |
| 5 | 0,0412 |
| 6 | 0,2111 |
| 7 | -0,1638 |
| 8 | -0,1913 |
| 9 | 0,3568 |
| 10 | 0,0387 |
| 11 | 0,0020 |
| 12 | 0,2814 |

The plot illustrating definition of the diagnosed system belonging to the condition 1 or 2 for the learning and control vectors is shown in Fig. 1.

## IV. CONCLUSION

Usage of the Vollterra-Wiener functionals for solving recognition of processes tasks is considered in this work. The indicated method has advantage, because it does not require any a priori knowledges of a testing system structure. Lacks of the considered method conclude large volume of evaluations for the kernels of polynomials describing high-order functionals $k \geq 2$ in (5). With aim to remove this difficulty it is possible to use the various approaches. For example, for reducing of a definition range of the polynomial functionals $G_k[h_k, x]$ (4) it is possible to use a property of a symmetry of the kernels $h_{kn}(i_1,...,i_n)$ concerning permutation of the arguments $i_1,...,i_n$ (for example, in (4) it is possible to put:

$$h_{kn}(i_1, i_2, ..., i_{n-1}, i_n) = h_{kn}(i_n, i_2, ..., i_{n-1}, i_1)).$$

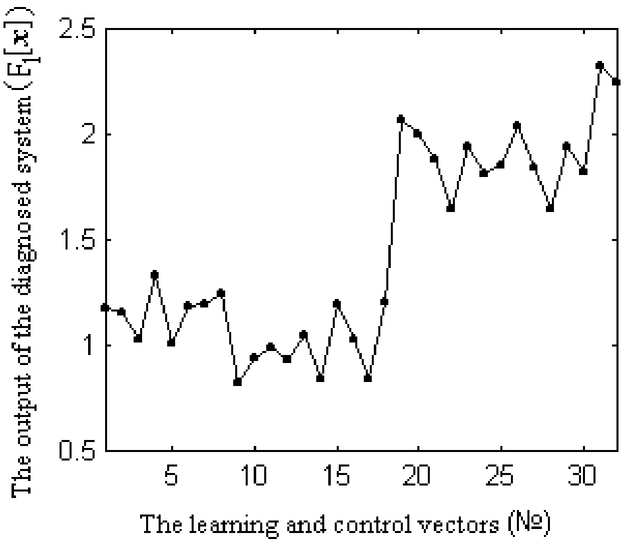


Fig. 1. The output of the recognition system on whose input the learning and control vectors have been acteed.

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